Extraction of the pion-nucleon sigma-term from the spectrum of exotic baryons

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The pion nucleon sigma-term is extracted on the basis of the soliton picture of the nucleon from the mass spectrum of usual and the recently observed exotic baryons, assuming that they have positive parity. The value found is consistent with that inferred by means of conventional methods from pion nucleon scattering data. The study can also be considered as a phenomenological consistency check of the soliton picture of baryons.

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I. INTRODUCTION

No experimental method is known to directly measure the pion-nucleon sigma-term $\sigma_{\pi N}$ [1, 2]. An indirect method consists in exploring a low energy theorem [3] which relates the value of the scalar-isoscalar form factor $\sigma(t)$ at the point $t = 2m_\pi^2$ to the isospin-even pion-nucleon scattering amplitude. Earlier analyses by Koch [4] and Gasser et al. [5] gave for $\sigma(2m_\pi^2)$ a value about 60 MeV, cf. Fig. 1. From the difference $\sigma(2m_\pi^2) - \sigma(0)$ found to be 15 MeV by Gasser et al. [5] one obtains for $\sigma_{\pi N} \equiv \sigma(0)$ a value about 45 MeV which was generally accepted until the late 1990's.

Recent analyses [6, 7, 8], however, tend to yield higher values for $\sigma(2m_\pi^2)$ in the range $(80-90)$ MeV, cf. Fig. 1, due to the impact of more recent and accurate data [9]. This results in a value of $\sigma_{\pi N}$ around 70 MeV. Such a large value of $\sigma_{\pi N}$ causes puzzles. According to a standard interpretation it implies a surprizingly large strangeness content of the nucleon (defined below in Eq. (26)), in contrast to what one would expect on the basis of the OZI-rule.

The precise knowledge of the value of $\sigma_{\pi N}$ is, however, of practical importance for numerous phenomenological applications. E.g., the value of $\sigma_{\pi N}$ enters the estimates of counting rates in searches of the Higgs boson [10], supersymmetric particles [11] or dark matter [12, 13]. Therefore independent and direct methods to access $\sigma_{\pi N}$ are welcome.

In this note we would like to draw the attention to a method relying on the soliton picture of the nucleon. The idea that baryons are different rotational excitations of the same object – a classical soliton of the chiral field – leads to numerous phenomenological relations among observables of different baryons, which are satisfied to a good accuracy and are model-independent, in the sense that they are due to symmetries of the soliton and do not depend on the dynamics of the respective model in which the soliton is realized.

For $\sigma_{\pi N}$ no such model-independent relation could be found. All one can do – sticking to known baryons – is to relate $\sigma_{\pi N}$ to mass splittings (among baryons in the SU(3)-flavour octet $J^P = \frac{1}{2}^+$) and the a priori unknown strangeness content of the nucleon [1, 2].

In other words, if one considers $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet baryons and explores soliton symmetries the information content is not sufficient to pin down $\sigma_{\pi N}$. The situation changes by including baryons from the next multiplet suggested by the soliton picture – the $\frac{1}{2}^+$ antidecuplet. After the prediction of its mass and width by Diakonov, Petrov and Polyakov [14] a candidate for the exotic "pentaquark" Θ^+, the lightest member of the antidecuplet, was observed by several groups [15, 16, 17, 18, 19, 20, 22, 23]. More recently also the finding of the second exotic baryon $\Xi_{3/2}^{++}$ was reported [21].

In fact, in the soliton picture of the nucleon in linear order in the strange quark mass the pion-nucleon sigma-term is unambiguously fixed in terms of mass splittings among baryons in the octet, decuplet and antidecuplet. Assuming that the exotic baryons Θ^+ and $\Xi_{3/2}$ are members of the antidecuplet allows to extract $\sigma_{\pi N}$ from the spectrum of usual and exotic baryons. The result compares well to the value of $\sigma_{\pi N}$ deduced from the more recent analyses of pion-nucleon scattering data. The quality and accuracy of such an extraction are discussed.

The note is organized as follows. In Sec. II $\sigma_{\pi N}$ is introduced. Sec. III contains a brief description of the soliton picture of baryons. In Sec. IV the relation between baryon mass splittings and $\sigma_{\pi N}$ is discussed. Sec. V contains the conclusions.

II. THE PION-NUCLEON SIGMA-TERM

The nucleon sigma-term form factor $\sigma(t)$ and the pion-nucleon sigma-term $\sigma_{\pi N}$ are defined as [1, 2]

$$\sigma(t)\,\bar{u}(\mathbf{p}')u(\mathbf{p}) = m\,\langle N'|\,(\bar{\psi}_u\psi_u + \bar{\psi}_d\psi_d)\,|N\rangle\,,$$

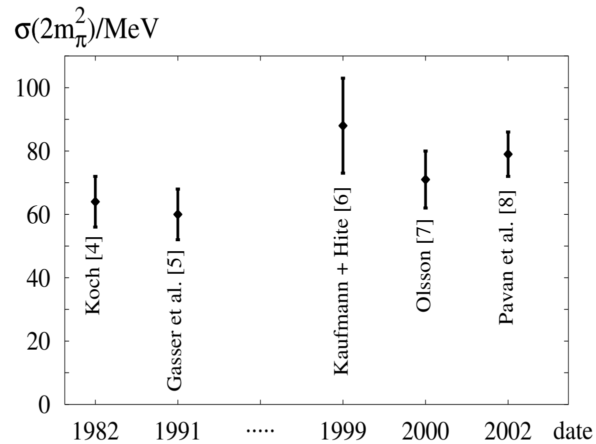

FIG. 1: The "historical development" of the value for $\sigma_{\pi N}$ in the last two decades (the time-axis is not linear).

$$\sigma_{\pi N} = \sigma(t)|_{t=0}\ , \quad t = (p-p')^2, \tag{1}$$

where $m = \frac{1}{2}(m_u+m_d)$ and the conventions are used $\langle N'|N\rangle = 2p^0\delta^{(3)}(\mathbf{p}-\mathbf{p'})$ and $\bar{u}(\mathbf{p})u(\mathbf{p}) = 2M_N$. Strictly speaking in Eq. (1) is neglected a "doubly isospin violating term" $\propto (m_u-m_d)(\bar\psi_u\psi_u - \bar\psi_d\psi_d)$.

The form factor $\sigma(t)$ is a normalization scale invariant quantity, which describes the elastic scattering off the nucleon due to the exchange of an isoscalar spin-zero particle. All that is known about it experimentally is its value at the so-called Cheng-Dashen point $t = 2m_\pi^2$. A low energy theorem [3] relates $\sigma(2m_\pi^2)$ to the isospin-even pion nucleon scattering amplitude, which can be inferred from pion-pion and pion-nucleon scattering data by means of dispersion relations. Earlier analyses by Koch in 1982 [4] and Gasser et al. in 1991 [5] gave, cf. Fig. 1,

$$\sigma(2m_\pi^2) = \begin{cases} (64\pm 8)\,\mathrm{MeV} & (1982)\ [4] \\ (60\pm 8)\,\mathrm{MeV} & (1991)\ [5]. \end{cases} \tag{2}$$

Gasser et al. [5] found from a dispersion relation analysis supplemented by chiral constraints

$$\sigma(2m_\pi^2) - \sigma(0) = (15.2\pm 0.4)\ \mathrm{MeV}\ , \tag{3}$$

which gave for $\sigma_{\pi N}$ a value about 45 MeV. Modern analyses yield a larger value for the form factor at the Cheng-Dashen point

$$\sigma(2m_\pi^2) = \begin{cases} (88\pm 15)\,\mathrm{MeV} & (1999)\ [6] \\ (71\pm 9)\,\mathrm{MeV} & (2000)\ [7] \\ (79\pm 7)\,\mathrm{MeV} & (2002)\ [8] \\ (80-90)\,\mathrm{MeV} & (2002)\ [9], \end{cases} \tag{4}$$

which can be explained by the impact the more recent and accurate data [9]. Thus recent analyses suggest

$$\sigma_{\pi N} \simeq (60-80)\,\mathrm{MeV}\ . \tag{5}$$

The analyses of pion-nucleon scattering data are involved and it is difficult to control the systematic error both, of the extractions of $\sigma(2m_\pi^2)$ and its connection to $\sigma_{\pi N}$ [4, 5, 6, 7, 8, 9]. However, there are no alternative methods to determine $\sigma_{\pi N}$.

The sum rule $\sigma_{\pi N} = m\int_0^1 \mathrm{d}x\,(e^u+e^d+e^{\bar u}+e^{\bar d})(x)$ due to Jaffe and Ji [28], which connects $\sigma_{\pi N}$ to the chirally odd twist-3 nucleon distribution function $e^a(x)$, is unfortunately useless as an alternative method to learn about $\sigma_{\pi N}$. On top of practical difficulties to access chirally odd (and twist-3) distribution functions in deeply inelastic scattering experiments [29], there is also a theoretical obstacle prohibiting such a "measurement" of $\sigma_{\pi N}$. The sum rule is saturated by a $\delta(x)$-type singularity. Such singularities can be (and were) "observed" in theoretical calculations [30, 31] but in experiment they can manifest themselves, in the best case, as a violation of the purely theoretical sum rule [32].

In lattice QCD – the most direct approach to QCD – the description of $\sigma_{\pi N}$ is (at present) challenging. Direct lattice calculations of $\sigma_{\pi N}$ meet the problem that the operator $\bar\psi\psi$ is not renormalization scale invariant [33]. An indirect method consists in exploring the Feynman-Hellmann theorem [34]

$$\sigma_{\pi N} = m\frac{\partial M_N}{\partial m} = m_\pi^2\frac{\partial M_N}{\partial m_\pi^2}\ , \tag{6}$$

to deduce $\sigma_{\pi N}$ from the pion mass dependence of the nucleon mass measured on the lattice [35, 36, 37]. In either case one faces the problem of extrapolating lattice data from presently $m_\pi \gtrsim 500\,\mathrm{MeV}$ down to the physical value of the pion mass which is subject to systematic uncertainties which are difficult to estimate. Results of extrapolations of most recent and accurate lattice data cover the range

$$\sigma_{\pi N} = (37^{+35}_{-13} - 73^{+15}_{-15})\,\mathrm{MeV} \tag{7}$$

depending on the extrapolation ansatz [38]. Chiral perturbation theory can in principle provide a rigorous guideline for the chiral extrapolation of lattice data – provided one is able to control the convergence of the chiral expansion up to $m_\pi \gtrsim 500\,\mathrm{MeV}$ which seems feasible [39, 40]. Chiral perturbation theory does not, however, allow to compute $\sigma_{\pi N}$ itself, which serves to absorb counter terms and has to be renormalized anew in each order of the chiral expansion.

The pion nucleon sigma-term was discussed in numerous models. See, e.g., [41, 42] for overviews of more recent works.

III. BARYONS IN THE SOLITON PICTURE

Since the early days of hadron physics symmetry principles have provided powerful guidelines for the

qualitative classification of hadrons and the quantitative understanding of the hadron mass spectrum.

In this context it is worthwhile to recall the relations derived by Gell-Mann and Okubo [43, 44] by considering SU(3) flavour symmetry and its breaking by quark mass terms up to linear order,

$$2M_{\rm N} + 2M_{\Xi} = 3M_{\Lambda} + M_{\Sigma} \;, \tag{8}$$

$$M_{\Delta} - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Xi^*} = M_{\Xi^*} - M_{\Omega} \;, \tag{9}$$

which are full-filled to within few percent. Historically Eq. (9) was used to predict the mass of the Ω^- baryon with impressive accuracy [45].

The Gell-Mann–Okubo formulae relate baryon masses within a multiplet, namely the octet in Eq. (8) and the decuplet in Eq. (9), cf. Figs. 1a and 1b. In order to relate masses from different multiplets one needs, however, more than the assumption of flavour symmetry. The limit of a large number of colours N_c – first discussed by 't Hooft [46] – provides further symmetry arguments.

Though in nature $N_c = 3$ seems not to be large the multi-colour limit yields numerous phenomenologically successful relations [47]. In particular the large N_c limit provides the basis for the picture of the nucleon as a classical soliton of the chiral pion field [48]. In the Skyrme model [49] or the chiral quark-soliton model [50] this picture is practically realized.

In these models the nucleon is a soliton of the pion field $U_{\rm SU(2)} = \exp(i\tau^a\pi^a)$ which is of the so-called hedgehog shape

$$\pi^a(\mathbf{x}) = \frac{{\rm x}^a}{|\mathbf{x}|}\, P\left(|\mathbf{x}|\right) \;, \tag{10}$$

such that flavour and space rotations become equivalent. Flavour SU(3) symmetry is considered by means of the following "embedding" ansatz

$$U_{\rm SU(3)} = \begin{pmatrix} U_{\rm SU(2)} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{pmatrix} \;. \tag{11}$$

In order to provide the classical soliton with spin, isospin and strangeness quantum numbers one has to consider the rotated field

$$U_{\rm SU(3)}(\mathbf{x},t) = R(t)\, U_{\rm SU(3)}(\mathbf{x})\, R^\dagger(t) \tag{12}$$

with $R(t)$ a time-dependent unitary SU(3) matrix. The quantization of the soliton rotation leads to the following rotational Hamiltonian and constraint

$$H_{\rm rot} = \frac{1}{2I_A}\sum_{a=1}^{3} J_a^2 + \frac{1}{2I_B}\sum_{a=4}^{7} J_a^2 \;, \quad J_8 = -\frac{N_c B}{2\sqrt{3}} \;. \tag{13}$$

In Eq. (13) the J_a $(a = 1,\, 2,\, \dots, 8)$ are the generators of the SU(3) group and I_A, I_B are moments of inertia characterizing the rotation of the soliton. The eigenfunctions of $H_{\rm rot}$ – the rotational baryon wave-functions with definite spin, isospin and strangeness quantum numbers – can be expressed in terms of Wigner finite-rotation matrices.

Of importance is the constraint of the generator J_8 in terms of the baryon number $B = 1$. In the Skyrme model it is due to the Wess-Zumino term [48, 53]. In the chiral quark soliton model it arises from a discrete bound state level in the spectrum of the single-quark Hamiltonian in the background of the static soliton field [55]. The consequence of this constraint is that only SU(3) multiplets containing particles with hypercharge $Y = 1$ are allowed. The lowest multiplets are the octet and decuplet of $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons respectively, cf. Figs. 1a and 1b.

In order to describe mass splittings within different multiplets it is necessary to introduce explicit chiral symmetry breaking by quark mass terms $\propto {\rm tr}\,\hat{m}(U-1)$ in the Skyrme model or $\psi\hat{m}\psi$ in the chiral quark-soliton model, where $\hat{m}$ is the SU(3) quark mass matrix with $m_u = m_d = 0$ and $m_s > 0$ in the following.

The exploration of the spin-flavour symmetry of the rotating soliton and its explicit breaking by linear quark masses terms yields relations among observables of different baryons, which are well satisfied in nature and model-independent – in the sense that they follow from symmetry considerations alone and do not depend on details of the dynamics, i.e. on how and in which theory the self-consistent field U is determined [51, 52, 53].

In particular one finds that (for $m_u = m_d = 0$) the eight different baryon masses in the octet and decuplet can be described in terms of 4 parameters:

- 2 parameters fix the mass splittings within the multiplets,
- 1 parameter characterizes the mass splitting between octet and decuplet,
- 1 parameter fixes the absolute mass scale for one multiplet, cf. [53].

These parameters can, of course, be computed in a specific model. However, what is more interesting in our context is to find general model-independent (in the above sense) relations, which allow to test phenomenologically the underlying idea of the soliton symmetry.

By eliminating the 4 parameters one obtains 4 relations among the eight baryon masses, namely the 3 Gell-Mann–Okubo formulae, Eqs. (8, 9), and in addition the Guadagnini relation [53]

$$8(M_{\Xi^*} - M_{\Sigma^*}) = 11M_{\Lambda} - 8M_N - 3M_{\Sigma} \,. \tag{14}$$

Eq. (14) relates mass splittings within different multiplets to each other. It is satisfied to an impressive accuracy of 1%.

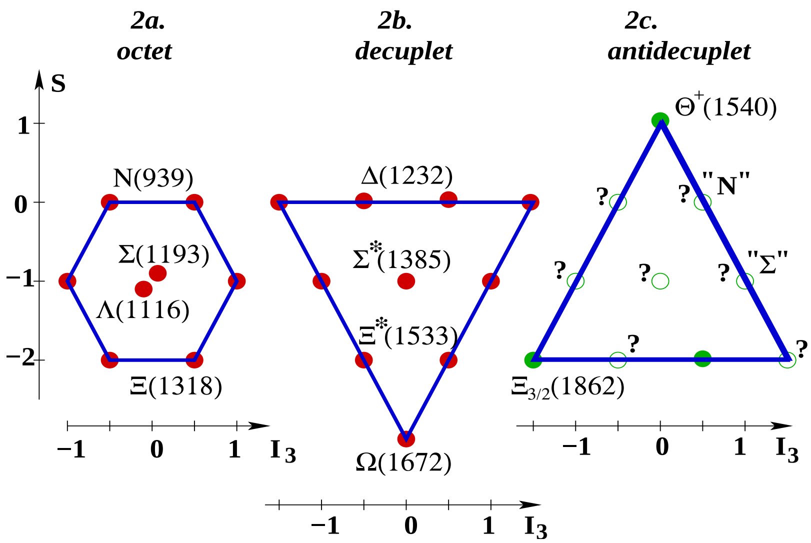

FIG. 2: Baryon multiplets. **a**. The $J^P = \frac{1}{2}^+$ octet. **b**. The $J^P = \frac{3}{2}^+$ decuplet. **c**. The $J^P = \frac{1}{2}^+$ antidecuplet predicted in the soliton picture of the nucleon with the two recently observed exotic candidates. The numbers in brackets denote the baryon masses (averaged over isospin where necessary) in MeV.

IV. EXOTIC BARYONS AND THE PION-NUCLEON SIGMA-TERM

The soliton symmetry as described by means of the rotational Hamiltonian (13) allows also higher multiplets. The next multiplet, after the octet and decuplet, is the $J^P = \frac{1}{2}^+$ antidecuplet, see Fig. 1c, which contains new "exotic" baryons.

From the point of view of the soliton picture there is nothing "unusual" about the baryons referred to as Θ^+ and $\Xi_{3/2}$. In a quark model, however, their quantum numbers can only be constructed by including an additional $\bar{q}q$ pair. Θ^+ has isospin zero and strangeness $S = 1$ which requires a combination $uudd\bar{s}$. The $\Xi_{3/2}^{--}$ member of the $\Xi_{3/2}$ isospin-quadruplet has $S = -2$ and $I_3 = -\frac{3}{2}$ which requires $\bar{u}ddss$, etc.

The other members of the antidecuplet, denoted here as $''\Sigma''$ and $''N''$, have "usual" (in the quark model language) quantum numbers. Candidates for these baryons were discussed in Ref. [25]. An unambiguous identifications of these states is difficult since mixings of the group theoretical states $|''N''\rangle$ and $|''\Sigma''\rangle$ with resonances of otherwise identical quantum numbers can occur. For our purposes it is important to note that to linear order in m_s such mixings do not effect the mass splittings within the octet and antidecuplet [25].

In the description of the masses of the antidecuplet (always to linear order of quark masses) two additional parameters appear:

- one characterizes the mass splittings,
- the other fixes the absolute mass scale.

The situation can be summarized as follows

$$
\begin{aligned}
M_N &= M_8 - 7A - B, \\
M_\Lambda &= M_8 - 4A, \\
M_\Sigma &= M_8 + 4A, \\
M_\Xi &= M_8 + 3A + B,
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
M_\Delta &= M_{10} - B, \\
M_{\Sigma^*} &= M_{10}, \\
M_{\Xi^*} &= M_{10} + B, \\
M_\Omega &= M_{10} + 2B,
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
M_{\Theta^+} &= M_{\overline{10}} - 2B + 2C, \\
M_{''N''} &= M_{\overline{10}} - B + C, \\
M_{''\Xi''} &= M_{\overline{10}}, \\
M_{\Sigma_{3/2}} &= M_{\overline{10}} + B - C,
\end{aligned}
\tag{17}
$$

where M_8, M_{10} and $M_{\overline{10}}$ characterize the average mass of the respective multiplet and A, B, C the splittings within the multiplets.

The 12 baryon masses can thus be expressed by means of 6 parameters. Eliminating these parameters one obtains in addition to the 3 Gell-Mann–Okubo and Guadagnini formulae, Eqs. (8, 9), two further relations. The new relations express an equal-mass splitting rule in the antidecuplet,

$$
M_{\Sigma_{3/2}} - M_{''\Xi''} = M_{''\Xi''} - M_{''N''} = M_{''N''} - M_{\Theta^+} \,, \tag{18}
$$

which is analogous to the relation (9) in the decuplet and was also observed in a description of pentaquarks in chiral perturbation theory [54]. Thus, neither the mass splitting in the new antidecuplet nor its absolute scale $M_{\overline{10}}$ can be fixed in terms of known baryon masses.

As observed by Diakonov, Petrov and Polyakov (this was actually an important ingredient in the prediction of Ref. [14]) the pion-nucleon sigma-term

can be expressed in terms of the same parameters, namely

$$\frac{m_s}{m}\,\sigma_{\pi N} = 3(35A + B + 4C)\ . \qquad (19)$$

At first glance one could be worried by the appearance of $m = \frac{1}{2}(m_u + m_d)$ in the denominator Eq. (19) since we work here in the chiral limit for light quarks $m_u = m_d = 0$. However, one has to recall that $\sigma_{\pi N}/m$ has a well-defined chiral limit – also in soliton models, see e.g. [41].

Eliminating the constants A, B, C in Eq. (19) one obtains

$$\begin{aligned}\frac{m_s}{m}\,\sigma_{\pi N} &= \underbrace{3(4M_\Sigma - 3M_\Lambda - M_N)}_{octet} + \underbrace{4(M_\Omega - M_\Delta)}_{decuplet} \\ &\quad - \underbrace{4(M_{\Xi_{3/2}} - M_{\Theta^+})}_{antidecuplet} \qquad (20)\end{aligned}$$

Thus, the soliton picture connects $\sigma_{\pi N}$ directly to the spectrum of baryons. In linear order of m_s the relation is simple but the prize to pay is that antidecuplet baryons are involved. In principle, if the antidecuplet would be established, the relation (20) would provide an attractive method to extract $\sigma_{\pi N}$.

Several comments are in order. In chiral perturbation theory *ratios* of quark masses can be considered as convention and scale independent quantities [57]. The framework of chiral perturbation theory eventually allows to express ratios of quark masses in terms of meson masses.

Eq. (20) follows from evaluating linear m_s effects in the soliton model. Therefore, for sake of consistency, it is preferable to use the value $m_s/m \equiv 2m_s/(m_u + m_d) = 25.9$ resulting from the consideration of chiral symmetry breaking effects to linear order in quark masses [56, 57]. Quadratic corrections yield $m_s/m = 24.4 \pm 1.5$ [57] – which is a small numerical change in view of the accuracy to which we work here.

Strictly speaking in the above discussion in the soliton model $m_s \neq 0$ but for light quarks the chiral limit was considered, i.e. $m = (m_u + m_d)/2 = 0$. Thus Eq. (20) gives the correct relation between $m_s \lim_{m\to 0} \sigma_{\pi N}/m$ on the left-hand-side and baryon mass splittings on the right-hand-side (in the limit $m \to 0$). If one wished to include finite-m effects one should consider also corrections due to $m_u \neq m_d$ and electromagnetic interactions on the same footing, which are of comparable magnitude. In principle the effect of such corrections can be minimized by considering particular linear combinations of masses from isospin multiplets instead of their averages as we do. However, for our purposes such corrections can be disregarded. When deducing $\sigma_{\pi N}$ from Eq. (20) we shall assume that the $\sigma_{\pi N}/m$ varies little in the chiral limit.

In the literature it is currently being debated whether the description of exotic baryons in the framework of soliton models can fully be justified in the large N_c-limit [58, 59, 60, 61, 62]. It was argued that – from the large N_c-limit point of view – a consistent description of multiplets containing exotics requires to go beyond the rotating soliton: For exotic multiplets vibrational modes may play an equally important role, in contrast to the usual octet and decuplet. Eq. (20) relates $\sigma_{\pi N}$ to mass splittings *within* multiplets. The rotating soliton description of mass splittings *within* multiplets could still be consistent with large N_c, e.g., when vibrational soliton modes were flavour independent or negligibly small with respect to the rotational zero modes. Then Eq. (20) would be consistent also from the large-N_c point of view. This issue, of course, deserves further investigations.

Eq. (20) can be rewritten by adding arbitrary multiples of the following "zeros"

$$\begin{aligned} 11M_\Lambda + 8(M_{\Sigma^*} - M_N - M_{\Xi^*}) - 3M_\Sigma &= 0\,, \quad (21)\\ 2M_{\Xi^*} - M_\Omega - M_{\Sigma^*} &= 0\,, \quad (22)\\ M_{\Xi^*} + M_{\Sigma^*} - M_\Omega - M_\Delta &= 0\,, \quad (23)\\ 3M_\Lambda - 2M_{\rm N} - 2M_\Xi + M_\Sigma &= 0\,, \quad (24)\end{aligned}$$

which result from Eqs. (8, 9, 14). Formally this would not change Eq. (20). In practice, however, the "zeros" are only approximate.

Eq. (21) is the most exact "zero", the right-hand-side (RHS) of Eq. (21) is 1 MeV if we insert baryon masses (averaged over isospin). Thus, if we added $25\times$ this "zero" we would change the value of $\sigma_{\pi N}$ by 1 MeV only. A common sense agreement could be to use Eq. (21) such that octet and decuplet (and uncertainties in their description) contribute to $\sigma_{\pi N}$ with comparable weight. Eq. (20) represents a possible choice – under the asthetical constraint to avoid awkward fractional coefficients.

Eqs. (22, 23) are less precise "zeros". The RHS of (22) is 9 MeV and the RHS of (23) yields 14 MeV. The uncertainty these relations introduce in Eq. (20) can be estimated by using instead of $4(M_\Omega - M_\Delta)$, e.g., $12(M_{\Sigma^*} - M_\Delta)$ or $12(M_\Omega - M_{\Xi^*})$. In this way one obtains (1750 ± 90) MeV for the contribution of the decuplet in Eq. (20). The RHS of Eq. (24) yields 27 MeV. We estimate the total contribution of the octet to Eq. (20) as (1455 ± 150) MeV.

Turning to the antidecuplet let us first point out that by choosing exotic antidecuplet members in Eq. (20) one avoids a principle complication, namely how to identify the non-exotic members in the new multiplet in view of possible complicated mixing patterns [25, 63]. (Recall that to linear order in m_s mixing does not effect mass differences *within* a multiplet.) Taking the candidates for Θ^+ and $\Xi_{3/2}$ for granted we obtain for the contribution of the antidecuplet in Eq. (20) the value (1288 ± 150) MeV

presuming that the mass splitting formula in the antidecuplet works no better and no worse than in other multiplets.

Thus, we obtain for the pion-nucleon sigma-term

$$\sigma_{\pi N} = (74 \pm 12)\,\mathrm{MeV}\,. \tag{25}$$

Alternatively, we can perform a best fit for the parameters A, B, C in Eqs. (15, 16) which gives respectively $(9 \pm 2)\,\mathrm{MeV}$, $(145 \pm 12)\,\mathrm{MeV}$, $(37 \pm 10)\,\mathrm{MeV}$. From (19) one then obtains $\sigma_{\pi N} = (71 \pm 14)\,\mathrm{MeV}$, in agreement with (25). (For completeness, the average masses of the multiplets are $M_8 = 1151\,\mathrm{MeV}$, $M_{10} = 1382\,\mathrm{MeV}$, $M_{\overline{10}} = 1754\,\mathrm{MeV}$.)

The result (25) is in reasonable agreement with the value of $\sigma_{\pi N}$ obtained from the recent dispersion relation analyses of pion nucleon scattering data, Eq. (5). It is also compatible with lattice results, Eq. (7).

Several comments are in order. Firstly, we took the candidates for Θ^+ and $\Xi_{3/2}$ for granted. However, in particular the $\Xi_{3/2}$ state has not yet been confirmed. Instead it was argued that the results of the NA49-experiment are in conflict with earlier experiments [65]. Secondly, we assumed that the soliton picture describes the antidecuplet to within the same accuracy as the octet and the decuplet, which can be checked only after all (also non-exotic) members of the antidecuplet will unambiguously be identified. Thirdly, the error in Eq. (25) reflects the accuracy to which the soliton picture describes the right-hand-side of Eq. (20), which does not necessarily comprise the entire uncertainty to which the soliton relation (20) *itself* is satisfied. The accuracy of (20) could be checked if we knew $\sigma_{\pi N}$ (and all antidecuplet masses) precisely.

Thus the error in (25) could be underestimated. However, this error does not appear unrealistic in view of the experience with other soliton relations – which connect, e.g., baryon-meson coupling constants [48], magnetic moments [66] or hyperon decay constants [67], and which typically hold to within an accuracy of $(10 - 20)\%$. (The Guadagnini formula (14) is another example.)

Finally, let us comment on the strangeness content of the nucleon which is defined as

$$y = \frac{2\langle N|\bar\psi_s\psi_s|N\rangle}{\langle N|\bar\psi_u\psi_u + \bar\psi_d\psi_d|N\rangle}\,. \tag{26}$$

The value of y can be inferred from mass splittings of the octet baryons. To linear order quark masses one obtains [1, 2]

$$y = 1 - \frac{m}{m_s - m}\,\frac{M_\Xi + M_\Sigma - 2M_N}{\sigma_{\pi N}}\,. \tag{27}$$

By means of Eq. (20) one can express y entirely in terms of baryon mass splittings – which yields $y \approx 0.6$. Inclusion of higher order quark mass terms tends to decrease the value of y [64] – which, however, still remains surprizingly large from the point of view of the OZI rule. The latter would imply the matrix element $\langle N|\bar\psi_s\psi_s|N\rangle$ to be small.

The term "strangeness content" is, however, somehow misleading. The scalar operator $\bar\psi_s\psi_s$ does not "count" strange quarks unlike the (zero component of the) vector operator $\bar\psi_s\gamma^\mu\psi_s$ does. Thus strictly speaking there is no *a priori* reason for the matrix element $\langle N|\bar\psi_s\psi_s|N\rangle$ to be small (apart from the OZI rule). In spite of a large strangeness content y the total contribution of strange quarks to the nucleon mass is reasonably small [68].

V. CONCLUSIONS

In the soliton picture of baryons in the linear treatment of strange quark mass terms the pion-nucleon sigma-term is simply related to the mass splittings in the octet, decuplet and antidecuplet [14]. Presuming that the Θ^+ and $\Xi^{--}_{3/2}$ exotic baryons [15, 16, 17, 18, 19, 20, 21, 22, 23] are members of the antidecuplet the pion-nucleon sigma-term was extracted from the mass splittings of usual and exotic baryons and found to be $\sigma_{\pi N} = 74\,\mathrm{MeV}$ with an accuracy of about $(15 - 20)\%$. This result is in good agreement with recent analyses of pion-nucleon and pion-pion scattering data which yield for the scalar-isoscalar form factor at the Cheng-Dashen point $\sigma(2m_\pi^2) = (80 - 90)\,\mathrm{MeV}$ [6, 7, 8, 9].

However, the present experimental basis for this analysis cannot be considered as solid. The $\Xi_{3/2}$ candidate has not yet been confirmed by independent groups, cf. Ref. [65] for a critical discussion. The widths are not measured directly [69], and in particular spin and parity of the exotic baryons are not established [26]. So it is not yet clear whether the exotic states fit into the soliton picture of the nucleon [14, 24] or into other approaches [70, 71, 72, 73].

If confirmed the soliton picture would provide an appealing method to access $\sigma_{\pi N}$ directly – with an uncertainty comparable to the accuracy to which Gell-Man–Okubo, and Guadagnini mass relations are satisfied. In future, with more information available on the antidecuplet, the uncertainty could be estimated more conservatively than it was possible here. This method could provide valuable information on $\sigma_{\pi N}$ supplementary to $\sigma(2m_\pi^2)$-extractions or direct lattice calculations. At the present stage the exercize presented here can be considered as a consistency check of the soliton picture – as it was presented along the lines of Ref. [25].

Further interesting issues are the inclusion of finite light quark current masses, isospin breaking effects

or higher order strange quark mass corrections by extending the methods elaborated in Ref. [55].

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